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Stochastic and nonlinear effects in semiconductor lasers

Professor Christina Masoller

**Universidad Politecnica de Cataluna
Department of Fisica e Ingenieria Nuclear
Calle de Colom 11
Terrassa, Spain 08222**

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14. ABSTRACT Semiconductor lasers are key elements in optical technologies, being coherent light sources in fiber optics communications, optical data storage, life sciences applications, material processing, and sensing. They have a huge economic impact and are crucial for the photonics technologies that improve our everyday life style. For developing the next generation of semiconductor lasers, more compact, faster, reliable and low-cost, is crucial to have a good understanding of the nonlinear light-matter interactions in semiconductor active media, and their nontrivial interplay with the various noise sources (such as spontaneous emission, thermal and electrical noise). Within the framework of this two-year project, detailed experimental and numerical studies have been performed, focusing on the interplay of noise and nonlinear dynamics. Specifically, we introduced a novel method of nonlinear time-series analysis, based on symbolic ordinal analysis, to characterize the temporal correlations of the intensity dropouts of a semiconductor laser with optical feedback operating in the low-frequency fluctuations regime.					
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Stochastic and nonlinear effects in semiconductor lasers
Reported period: 15 June 2010 – 14 June 2012

Principal Investigator: Cristina Masoller*

Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya (UPC), Colom 11, E-08222 Terrassa, Spain

(Dated: September 15, 2012)

1. SUMMARY

This Final Report summarizes our main achievements during the two years of this project. Our work was aimed at advancing the present understanding of the interplay of nonlinearities and stochastic effects in semiconductor lasers. Semiconductor lasers are key elements in optical technologies, being coherent light sources in fiber optics communications, optical data storage, life sciences applications, material processing, and sensing. They have a huge economic impact and are crucial for the photonics technologies that improve our everyday life style. For developing the next generation of semiconductor lasers, more compact, faster, reliable and low-cost, is crucial to have a good understanding of the nonlinear light-matter interactions in semiconductor active media, and their nontrivial interplay with the various noise sources (such as spontaneous emission, thermal and electrical noise). Within the framework of this two-year project, detailed experimental and numerical studies have been performed, focusing on the interplay of noise and nonlinear dynamics. Specifically, we introduced a **novel method of nonlinear time-series analysis**, based on symbolic ordinal analysis, to characterize the temporal correlations of the intensity dropouts of a semiconductor laser with optical feedback operating in the low-frequency fluctuations regime. The ordinal method was successfully tested and can be employed for the analysis of other noisy experimental data at an event-level description, such as interspike-intervals in neuronal recordings, where weak signatures of deterministic nonlinear dynamics can be obscured by the presence of noise. Another research focus in this project was the generation of **all-optical square-waves** with switching times in the nanosecond time-scale. We considered two setups capable of generating all-optically square-waves: a semiconductor laser with orthogonal feedback and two semiconductor lasers mutually and orthogonally coupled. In both setups we considered two types of semiconductor lasers, edge-emitting lasers (EELs) and vertical-cavity surface emitting lasers (VCSELs), since they display different polarization dynamics. We found optimal operating conditions for generating regular and stable square-wave polarization switching with a periodicity fully controlled by the feedback delay time or by the mutual coupling delay time. A third research topic in this project were **extreme events** in the form of ultra-high intensity pulses observed experimentally and numerically in the output intensity of a semiconductor laser with continuous-wave external optical injection. We showed that these ultra-high intensity pulses could be interpreted as **optical rogue waves**, that they can be predicted with long anticipation time, that they are enabled by a crisis-like process, and that noise can be employed to either enhance or suppress their probability of occurrence. By providing a good understanding of the mechanisms triggering and controlling the rogue waves, our results can contribute to improve the performance of injected lasers, and can also enable new experiments to test if these mechanisms are also involved in other natural systems where rogue waves have been observed. The results obtained in the framework of this two-year project were published in 11 high-impact journal papers, and were presented as Invited Talks and Oral/Poster contributions in several international conferences and workshops. A PhD student partially funded by this project, Jordi Zamora Munt, finished his PhD at the Universitat Politècnica de Catalunya (UPC, Spain) last June 2011.

2. PUBLICATIONS

The results of our research were published in 11 papers in high-impact journals in the fields of photonics and nonlinear physics: two Physical Review Letters (impact factor 7.37), two Optics Express (impact factor 3.587), one Optics Letters (impact factor 3.399), three Physical Review A (impact factor 2.878), one Physical Review E (impact factor 2.255), one IEEE J. Quantum Electronics (impact factor 1.879) and one European Journal of Physics D (impact factor 1.476).

*Electronic address: cristina.masoller@upc.edu, cristina.masoller@gmail.com

1. J. Zamora-Munt, C. Masoller, J. Garca-Ojalvo and R. Roy, *Crowd synchrony and quorum sensing in delay-coupled lasers*, Phys. Rev. Lett. 105, 264101 (2010).
2. C. Bonatto, M. Feyereisen, S. Barland, M. Giudici, C. Masoller, J. R. Rios Leite and J. R. Tredicce, *Deterministic optical rogue waves*, Phys. Rev. Lett. 107, 053901 (2011).
3. J. Zamora-Munt and C. Masoller, *Numerical implementation of a VCSEL-based stochastic logic gate via polarization bistability*, Opt. Express 18, 16418-16429 (2010).
4. M. S. Torre, A. Gavrielides and C. Masoller, *Numerical characterization of transient polarization square-wave switching in two Orthogonally Coupled VCSELs*, Opt. Express 19, 20269 (2011).
5. Y. Hong, C. Masoller, M. S. Torre, S. Priyadarshi, A. A. Qader, P. S. Spencer and K. A. Shore, *Thermal effects and dynamical hysteresis in the turn-on and turn-off of vertical-cavity surface-emitting lasers*, Opt. Lett. 35, 3688-3690 (2010).
6. J. Zamora-Munt, C. Masoller and J. Garcia-Ojalvo, *Transient low-frequency fluctuations in semiconductor lasers with optical feedback*, Phys. Rev. A 81, 033820 (2010).
7. J. Tiana-Alsina, M. C. Torrent, O. A. Rosso, C. Masoller and J. Garcia-Ojalvo, *Quantifying the statistical complexity of low-frequency fluctuations in semiconductor lasers with optical feedback*, Phys. Rev. A 82, 013819 (2010).
8. C. Masoller, D. Sukow, A. Gavrielides and M. Sciamanna, *Bifurcation to square-wave switching in orthogonally delay-coupled semiconductor lasers: theory and experiment*, Phys. Rev. A 84, 023838 (2011).
9. N. Rubido, J. Tiana-Alsina, M. C. Torrent, J. Garcia-Ojalvo and C. Masoller, *Language organization and temporal correlations in the spiking activity of an excitable laser: Experiments and model comparison*, Phys. Rev. E 84, 026202 (2011).
10. M. S. Torre and C. Masoller, *Dynamical hysteresis and thermal effects in vertical-cavity surface-emitting lasers*, IEEE J. Quantum Electron. 46, 1788-1793 (2010).
11. C. Masoller and M. Oria, *Frequency dynamics of semiconductor lasers with atomic absorbers: theory and experiments*, Eur. Phys. J. D 58, 191-196 (2010).

3. RESEARCH TOPICS

Our research continued the work done in the framework of our previous AFOSR three-year project (FA9550-07-1-0238, 2007-2009) and focused on four main topics:

1. Polarization dynamics of vertical-cavity surface emitting lasers (VCSELs)
2. Nonlinear dynamics of optically injected VCSELs
3. Dynamics of semiconductor lasers with time-delayed optical feedback
4. Dynamics of semiconductor lasers with time-delayed mutual coupling

In the next subsections we present the main achievements in each topic and the related publications.

3.1. Polarization dynamics of vertical-cavity surface emitting lasers (VCSELs)

Main achievements:

- Numerical demonstration of an optoelectronic implementation of a VCSEL-based stochastic logic gate via the interplay of i) polarization bistability, ii) current modulation and iii) spontaneous emission noise.
- Experimental and numerical demonstration of thermally induced negative dynamical hysteresis in the turn-on and turn-off of VCSELs.

Related publications:

- J. Zamora-Munt and C. Masoller, *Numerical implementation of a VCSEL-based stochastic logic gate via polarization bistability*, Opt. Express 18, 16418-16429 (2010).
- Y. Hong, C. Masoller, M. S. Torre, S. Priyadarshi, A. A. Qader, P. S. Spencer and K. A. Shore, *Thermal effects and dynamical hysteresis in the turn-on and turn-off of vertical-cavity surface-emitting lasers*, Opt. Lett. 35, 3688-3690 (2010).
- M. S. Torre and C. Masoller, *Dynamical hysteresis and thermal effects in vertical-cavity surface-emitting lasers*, IEEE J. Quantum Electron. 46, 1788-1793 (2010).

3.2. Nonlinear dynamics of optically injected VCSELs

Main achievement:

- Experimental and numerical demonstration of deterministic rogue waves in the output intensity of a VCSEL with cw optical injection.

Related publications:

- C. Bonatto, M. Feyereisen, S. Barland, M. Giudici, C. Masoller, J. R. Rios Leite and J. R. Tredicce, *Deterministic optical rogue waves*, Phys. Rev. Lett. 107, 053901 (2011).

3.3. Dynamics of semiconductor lasers with time-delayed optical feedback

Main achievements:

- Numerical study of transient low-frequency fluctuations (LFFs); identification of the key parameters that affect the duration of the transient dynamics.
- Experimental characterization of the complexity of the LFFs dynamics employing the novel method of nonlinear time-series analysis referred to as ordinal analysis.
- Experimental and numerical characterization of the frequency dynamics of a semiconductor laser with an atomic absorber placed in the external cavity.

Related publications:

- J. Zamora-Munt, C. Masoller and J. Garcia-Ojalvo, *Transient low-frequency fluctuations in semiconductor lasers with optical feedback*, Phys. Rev. A 81, 033820 (2010).
- J. Tiana-Alsina, M. C. Torrent, O. A. Rosso, C. Masoller and J. Garcia-Ojalvo, *Quantifying the statistical complexity of low-frequency fluctuations in semiconductor lasers with optical feedback*, Phys. Rev. A 82, 013819 (2010).
- N. Rubido, J. Tiana-Alsina, M. C. Torrent, J. Garcia-Ojalvo, and C. Masoller, *Language organization and temporal correlations in the spiking activity of an excitable laser: Experiments and model comparison*, Phys. Rev. E 84, 026202 (2011).
- C. Masoller and M. Oria, *Frequency dynamics of semiconductor lasers with atomic absorbers: theory and experiments*, Eur. Phys. J. D 58, 191-196 (2010).

3.4. Dynamics of semiconductor lasers with time-delayed mutual coupling

Main achievements:

- Numerical demonstration of the phenomena of crow synchrony and quorum sensing in star-coupled lasers.
- Experimental and numerical demonstration of regular square-wave switching in orthogonally mutually coupled EELs; numerical demonstration of stable switching in narrow parameter regions depending on the gain self- and cross-saturation coefficients and the frequency detuning between the two polarizations.

- Numerical characterization of transient square-wave switching in orthogonally time-delayed mutually coupled VCSELs.

Related publications:

- J. Zamora-Munt, C. Masoller, J. Garca-Ojalvo and R. Roy, *Crowd synchrony and quorum sensing in delay-coupled lasers*, Phys. Rev. Lett. 105, 264101 (2010).
- C. Masoller, D. Sukow, A. Gavrielides and M. Sciamanna, *Bifurcation to square-wave switching in orthogonally delay-coupled semiconductor lasers: theory and experiment*, Phys. Rev. A 84 023838 (2011).
- M. S. Torre, A. Gavrielides and C. Masoller, *Numerical characterization of transient polarization square-wave switching in two Orthogonally Coupled VCSELs*, Opt. Express. 19, 20269 (2011).

4. PHD THESIS

TITLE: Nonlinear and Stochastic Dynamics of Semiconductor Lasers: Modulation, Transient Dynamics and Synchronization

STUDENT: Jordi Zamora Munt

DIRECTOR(S): C. Masoller, J. Garcia Ojalvo

UNIVERSITY: Universitat Politecnica de Catalunya

FACULTY/SCHOOL: Departament de Física i Enginyeria Nuclear

YEAR: 2011

MARK: Cumun Laudae

5. SCIENCE COMMUNICATION

5.1. Invited talks

1. 7th European Nonlinear Dynamics Conference (ENOC 2011), Rome, Italy, July 2011,
Polarization square-wave switching in orthogonally delay-coupled semiconductor lasers
Invited talk presented by C. Masoller at the symposium on Time Delayed Systems
2. Nonlinear Physics and Applications (NOLPA 2011), Joao Pessoa, Brasil, September 2011,
Rogue Waves and Square Waves in the Nonlinear Dynamics of Semiconductor Lasers
Invited talk presented by C. Masoller
3. Photonics Europe, Brussels, Belgium, April 2012,
Square-wave switching in orthogonal polarization delay-coupled semiconductor lasers
Invited talk presented by M. Sciamanna
4. Experimental Chaos and Complexity, Ann Arbor, Mi, US, May 2012,
Deterministic optical rogue waves
Invited talk presented by J. R. Rios Leite
5. Workshop on Delayed Complex Systems, Mallorca, Spain, June 2012,
Exploiting bistability, time delay and noise for obtaining all-optically square-wave switching
Invited talk presented by C. Masoller

5.2. Oral contributions

1. 5th Rio de la Plata Workshop on Laser Dynamics and Nonlinear Photonics, Colonia, Uruguay, December 2011,
Polarization square-wave switching in orthogonally delay-coupled semiconductor lasers
Contributed talk presented by C. Masoller
2. Dynamics Days, January 2012, Baltimore, US, January 2012,
Deterministic optical rogue waves
Contributed talk presented by J. R. Rios Leite

5.3. Poster presentations

1. Nolineal 2012, Zaragoza, Spain, June 2012,
Distinguishing determinism from stochasticity in the spiking activity of semiconductor lasers with feedback using ordinal time-series analysis
Poster presented by A. Aragoneses
2. Workshop on Delayed Complex Systems, Mallorca, Spain, June 2012,
Distinguishing determinism from stochasticity in the spiking activity of semiconductor lasers with feedback using ordinal time-series analysis
Poster presented by N. Rubido
3. International Conference on Transparent Optical Networks (ICTON), Warwick, UK, July 2012,
All optical implementation of a stochastic logic gate using a VCSEL with external optical injection
Poster presented by S. Perrone

5.4. Book chapters

1. J. Zamora-Munt, C. Masoller and J. Garca-Ojalvo, “Multi-stability and transient chaotic dynamics in semiconductor lasers with time-delayed optical feedback” (pp. 78-83).
Book title: *From physics to control through an emergent view*, World Scientific Series on Nonlinear Science, Series B Vol. 15.
Editors: Luigi Fortuna, Alexander Fradkov and Mattia Frasca.
Published by: World Scientific Publishing (2010). ISBN: 9789814313148
2. J. Zamora-Munt and C. Masoller, “Exploiting Noise and Polarization Bistability in Vertical-Cavity Surface-Emitting Lasers for Fast Pulse Generation and Logic Operations”.
Book title: *Nonlinear Laser Dynamics: From Quantum Dots to Cryptography*.
Editor: Kathy Ludge.
Published by: Wiley-VCH Verlag GmbH & Co. KGaA (2012). ISBN: 3527411003

5.5. Other publications

Our work on deterministic optical rogue waves (Phys. Rev. Lett. 107, 053901, 2011) was featured in Research Highlights of *Nature Photonics* (Vol. 5, No. 10, Page 571 DOI:10.1038/nphoton.2011.240) and in *Optics and Photonics News* (February 2012, page 6).

6. COLLABORATORS AND PEOPLE INVOLVED

The work carried out during this two-year project was done in collaboration with several well-known research groups working in the field of semiconductor laser nonlinear dynamics. Our work on square-wave regular switching was done in collaboration with Dr. Tom Gavrielides (EOARD), Dr. David Sukow (Washington and Lee University, US), Dr. Marc Sciamanna (Supelec, Metz, France), and Dra. Maria Susana Torre (UNCPBA, Tandil, Argentina): the experiments were carried out at Dr. Sukow’s lab while numerical simulations and analytical studies were done by Drs. Gavrielides, Masoller, Sciamanna and Torre.

The statistical study of the LFF power dropouts employing symbolic ordinal analysis was done in collaboration with several members of the research group in Terrassa UPC (Drs. Jordi G. Ojalvo and M. C. Torrent, and two PhD students, J. Tiana and N. Rubido). Our work on optical rogue waves was done in collaboration with the research group at the Institut Non-Lineaire de Nice, France (Drs. Stephane Barland, Massimo Giudici, and Jorge Tredicce), where the experiments were done. Simulations were done by C. Masoller and a postdoctoral researcher, Dr. C. Bonatto, partially funded by this project. Dr. Rios Leite (Universidade Federal de Pernambuco, Recife, Brazil) also participated of the discussions and analysis of the results.

In addition, the study of the synchronization properties of star-coupled lasers, the simulations were done by the PhD student Jordi Zamora Munt, partially funded by this project, in collaboration with Prof. Raj Roy (University of Maryland, US) and Jordi G. Ojalvo (UPC).

Regarding our work on hysteresis phenomena in VCSELs, the experimental work was done at the lab of the research group in Bangor University, UK (Prof. Alan Shore, Dr. Yanhua Hong) and the numerical simulations were done in collaboration with Dra. Maria Susana Torre (UNCPBA, Tandil, Argentina).

7. MODELS EMPLOYED

7.1. Model for edge-emitting lasers

The model used to study the dynamics of two EELs mutually coupled such that the x polarization of one laser is rotated 90 degrees and is injected into the y polarization of the other laser is:

$$\frac{dE_{x,i}}{dt} = k(1 + j\alpha)(g_{x,i} - 1)E_{x,i} + \sqrt{\beta_{sp}}\xi_{x,i}, \quad (1)$$

$$\frac{dE_{y,i}}{dt} = (j\delta)E_{y,i} + k(1 + j\alpha)(g_{y,i} - 1 - \beta)E_{y,i} + \eta E_{x,3-i}(t - \tau)e^{-j\omega_0\tau} + \sqrt{\beta_{sp}}\xi_{y,i}, \quad (2)$$

$$\frac{dN_i}{dt} = \gamma_N[\mu - N_i - g_{x,i}I_{x,i} - g_{y,i}I_{y,i}]. \quad (3)$$

Here $i = 1$ and $i = 2$ denote the two lasers, E_x and E_y are orthogonal linearly polarized slowly-varying complex amplitudes and N is the carrier density. The coupling of each laser x polarization into the other laser y polarization is described with time-delayed terms in Eq. (2). In the absence of optical coupling the emission frequency of the two lasers is the same (ω_0), which is the frequency of the x polarization and is taken as the reference frequency.

The model includes a frequency detuning between the x and the y polarizations, represented by the parameter δ , and self- and cross-saturation coefficients:

$$g_{x,i} = \frac{N_i}{1 + \epsilon_{xx}I_{x,i} + \epsilon_{xy}I_{y,i}}, \quad (4)$$

$$g_{y,i} = \frac{N_i}{(1 + \epsilon_{yx}I_{x,i} + \epsilon_{yy}I_{y,i})}. \quad (5)$$

Other model parameters are: k is the field decay rate, γ_N is the carrier decay rate, α the linewidth enhancement factor, β is the linear loss anisotropy, β_{sp} is the noise strength, $\xi_{x,y}$ are uncorrelated Gaussian white noises and μ is the injection current parameter, normalized such that the solitary threshold is at $\mu_{th,s} = 1$. The coupling parameters are η and τ , which represent the coupling strength and the delay time respectively.

7.2. Models for vertical-cavity surface-emitting lasers

7.2.1. Model for a VCSEL with PR feedback

The model used to describe the dynamics of a VCSEL with polarization-rotated (PR) optical feedback, such that one polarization is selected, is rotated by 90 degrees and then is reinjected into the laser, is:

$$\frac{dE_x}{dt} = k(1 + j\alpha)[(N - 1)E_x + jnE_y] - (\gamma_a + j\gamma_p)E_x + \sqrt{\beta_{sp}}\xi_x(t) + \eta_y E_y(t - \tau), \quad (6)$$

$$\frac{dE_y}{dt} = k(1 + j\alpha)[(N - 1)E_y - jnE_x] + (\gamma_a + j\gamma_p)E_y + \sqrt{\beta_{sp}}\xi_y(t) + \eta_x E_x(t - \tau), \quad (7)$$

$$\frac{dN}{dt} = -\gamma_n[-\mu + N(1 + |E_x|^2 + |E_y|^2) + jn(E_y E_x^* - E_x E_y^*)], \quad (8)$$

$$\frac{dn}{dt} = -\gamma_s n - \gamma_N[n(|E_x|^2 + |E_y|^2) + jN(E_y E_x^* - E_x E_y^*)]. \quad (9)$$

Here, E_x and E_y are orthogonal linearly polarized field amplitudes, N and n are two carrier densities ($N = N_+ + N_-$, $n = N_+ - N_-$ with N_+ and N_- being carrier populations with opposite spin), k is the field decay rate, γ_n is the carrier decay rate, γ_s is the spin-flip rate, α the linewidth enhancement factor, γ_a and γ_p are anisotropies representing dichroism and birefringence: for $\gamma_a > 0$ ($\gamma_p > 0$) the y polarization has a lower threshold (a higher frequency) than the x polarization. μ is the injection current parameter, normalized such that the threshold of the solitary laser in the

absence of anisotropies is at $\mu_{th,s} = 1$, β_{sp} is the strength of spontaneous emission noise, and $\xi_{x,y}$ are uncorrelated Gaussian white noises.

The feedback parameters are the injection strength, η , and the delay time, τ . We considered two types of polarization-rotated optical feedback: (i) $x \rightarrow y$ (the x polarization is selected and then rotated), for which $\eta_x = \eta$, $\eta_y = 0$ and (ii) $y \rightarrow x$ (the y polarization is selected and then rotated), for which $\eta_x = 0$, $\eta_y = \eta$. Because of birefringence and the α factor, these two types of PR feedback are not symmetric.

7.2.2. Model for two VCSELs with PR mutual coupling

The model used to describe the dynamics of two VCSELs with polarization-rotated (PR) mutual coupling is:

$$\frac{dE_{ix}}{dt} = k(1 + j\alpha)[(N_i - 1)E_{ix} + jn_i E_{iy}] - (\gamma_a + j\gamma_p)E_{ix} + \sqrt{\beta_{sp}}\xi_x(t) + \eta_y E_{(3-i)y}(t - \tau), \quad (10)$$

$$\frac{dE_{iy}}{dt} = k(1 + j\alpha)[(N_i - 1)E_{iy} - jn_i E_{ix}] + (\gamma_a + j\gamma_p)E_{iy} + \sqrt{\beta_{sp}}\xi_y(t) + \eta_x E_{(3-i)x}(t - \tau), \quad (11)$$

$$\frac{dN_i}{dt} = -\gamma_n[-\mu + N_i(1 + |E_{ix}|^2 + |E_{iy}|^2) + jn_i(E_{iy}E_{ix}^* - E_{ix}E_{iy}^*)], \quad (12)$$

$$\frac{dn_i}{dt} = -\gamma_s n_i - \gamma_n[n_i(|E_{ix}|^2 + |E_{iy}|^2) + jN_i(E_{iy}E_{ix}^* - E_{ix}E_{iy}^*)]. \quad (13)$$

Here $i = 1$ and $i = 2$ denote the two lasers, the variables and the parameters have the same meaning as in Eqs. (6)-(9). The coupling strengths, η_x and η_y , are such that, when the x polarization of one laser is injected into the y polarization of the other laser ($x \rightarrow y$), $\eta_x = \eta$ and $\eta_y = 0$, and when the y polarization of one laser is injected into the x polarization of the other laser ($y \rightarrow x$), $\eta_x = 0$ and $\eta_y = \eta$.